Derivation of proportional partial fixity factors

Consider a beam element length L, elasticity E, second moment of area I, connection stiffness Ka same at both ends. From K H Gerstle: *Effect of connections on frames*, Journal of Constructional Steel Research vol 10 1988 p 250:-

Fixed end moments: M1 = M2
= w*L*L*/{12*[1 + (2*E*I/(Ka*L))]}

If fully fixed, Ka = infinite, M1 = M2 = w*L*L/12

If pinned, Ka = zero, M1 = M2 = 0.

So proportion of full fixity factor: Kf = 1/[1 + (2*E*I/(Ka*L))] (1)

Alternatively, relating absolute fixity Ka to basic fixed member stiffness 4*E*I/L

Km = Ka*L / (4*E*I) (2)

Which can be transposed: Ka = 4*E*I*Km/L (3)

Note that some authorities relate proportional stiffnesses to EI (Gerstle) or to 3EI/L (member pinned at the far end - Cunningham). This just means different values for partial stiffnesses but the extreme values for Kf are the same (0 and 1.0).

Substitute (2) in (1) to get relation between proportional member stiffness Km and proportional full stiffness Kf :-

Kf = 1/{1 + (0.50/Km)}

= Km/{0.50 + Km} (4)

Which can be transposed: Km = 0.50*Kf / (1 – Kf) (5)

Note that above expressions 1, 2, 3 assume consistent units.

For L in m, I in cm4, E in kN/mm2, and Ka in kNm/radian:-

Km = Ka*L / (4*E*I) (2)

= kNm*m*mm*mm*10^8

4*kN*cm*cm*cm*cm*10^6

= 100*Ka*L / (4*E*I)

= 25*Ka*L / (E*I)